Geometry Notes

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MATH

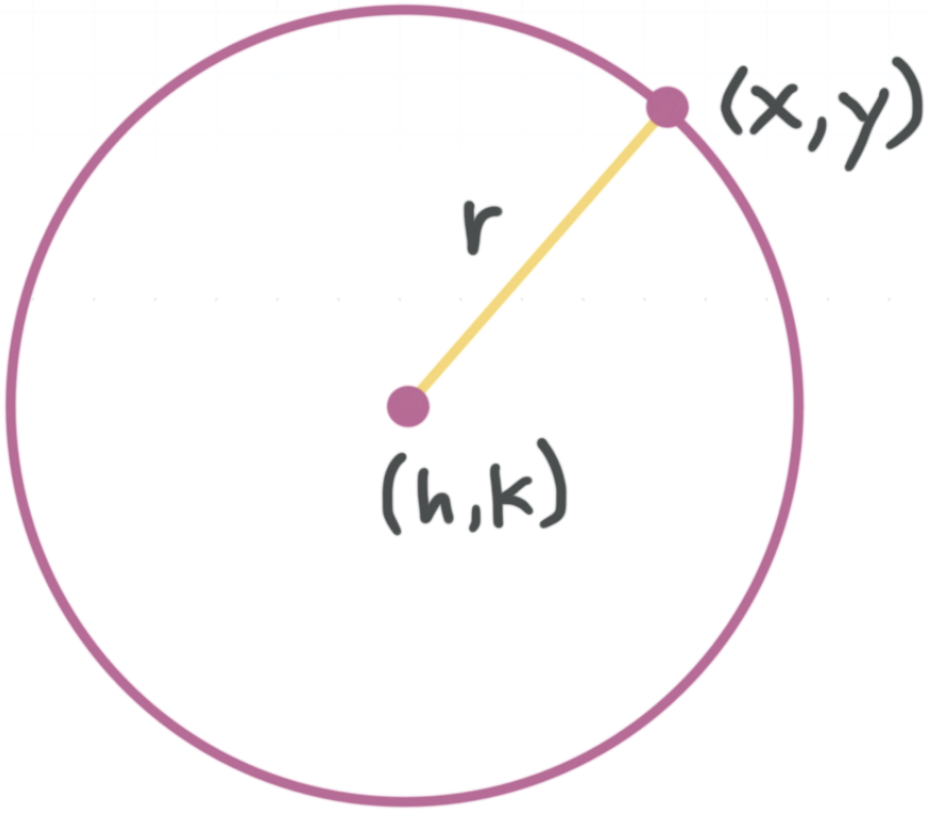
Equation of a circle

In this lesson we'll look at the equation for a circle, $(x-h)^2+(y-k)^2=r^2$, and how to use it to graph a circle, interpret points on a circle and write an equation given a graph or the special features of a circle.

A circle can be defined by a center point and a radius of a certain length. In the equation of a circle

$$(x-h)^2+(y-k)^2=r^2$$

the center is called (h,k) and the radius is r. From the equation, you can see that the circle is the collection of all the (x,y) points that are a distance r from the center, (h,k).



The collection of (x,y) values aren the points on the circle's circumference. Let's start by working through an example.

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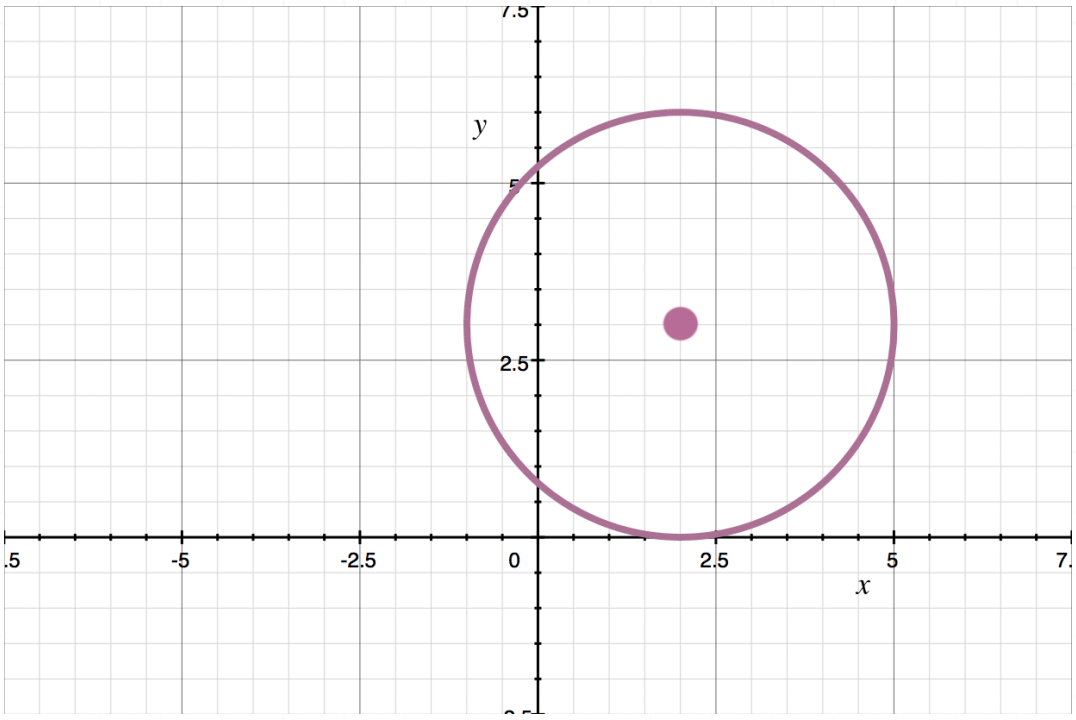
Example

What is the equation of the circle?



We need to find the equation of a circle in the form $(x-h)^2+(y-k)^2=r^2$, which means we need to find the center point and the length of the radius.

The center is at $(2,3)$, so $h=2$ and $k=3$.

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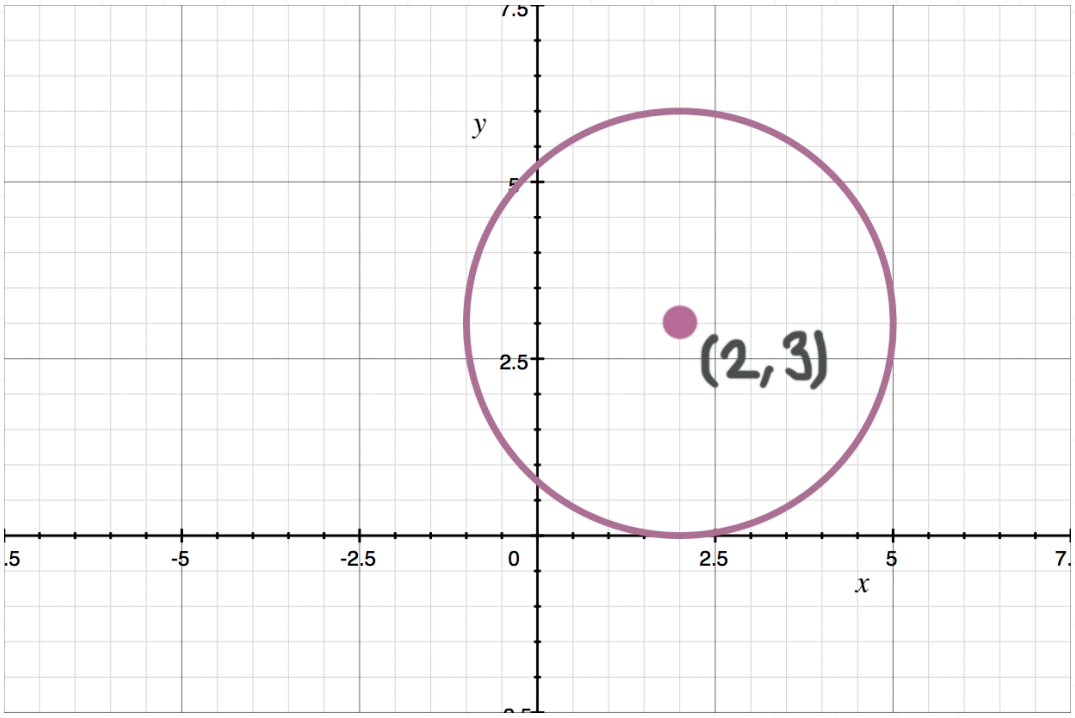
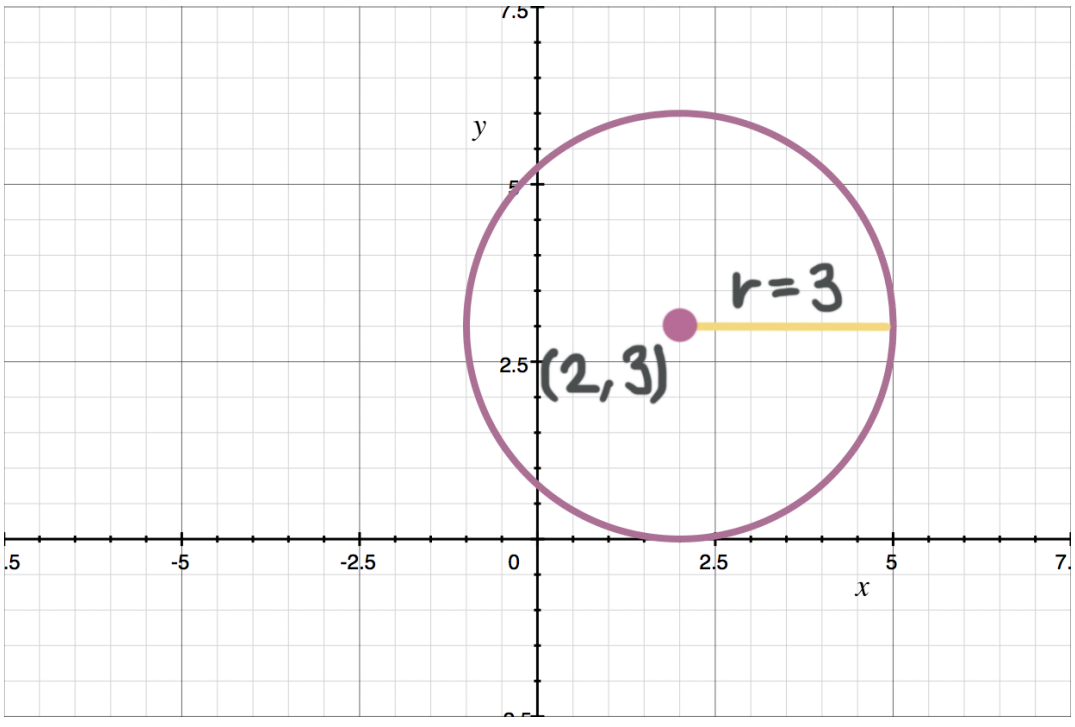


Now let's count from the center to a point on the circumference to find the length of the radius.



The radius is 3 so $r = 3$. Now let's plug everything into the standard form of a circle.

$$(x-2)^2 + (y-3)^2 = 3^2$$

$$(x-2)^2 + (y-3)^2 = 9$$

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Sometimes we want to know the center and radius of a circle given the equation of the circle.

Example

What is the center and radius of the circle?

$$x^2 + (y - 3)^2 = 27$$

We can rewrite the equation as

$$(x - 0)^2 + (y - 3)^2 = 27$$

Which lets us identify $h = 0$ and $k = 3$, so the center is at $(0,3)$. And the radius is then $\sqrt{27}$, so

$$r = \sqrt{27}$$

$$r = \sqrt{9 \cdot 3}$$

$$r = \sqrt{9} \cdot \sqrt{3}$$

$$r = 3\sqrt{3}$$

Sometimes we want to know the x-intercepts of a circle.

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Example

What are the x-intercepts of the circle?

$$(x-2)^2+(y+1)^2=16$$

At the x-intercepts, $y=0$, so let y be 0, and rewrite the equation.

$$(x-2)^2+(y+1)^2=16$$

$$(x-2)^2+(0+1)^2=16$$

$$(x-2)^2+1^2=16$$

$$x^2-4x+4+1=16$$

$$x^2-4x+5=16$$

$$x^2-4x-11=0$$

If you can't factor the equation to solve for x you can use the quadratic formula. In this case, $a=1$, $b=-4$, and $c=-11$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{4\pm\sqrt{(-4)^2-4(1)(-11)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{60}}{2}=\frac{4\pm\sqrt{4\cdot 15}}{2}=\frac{4\pm 2\sqrt{15}}{2}=2\pm\sqrt{15}$$

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These are the x-intercepts.

Sometimes to find out information about a circle you'll need to know how to complete the square.

Example

Find the center and radius of the circle.

$$x^2 + y^2 + 24x + 10y + 160 = 0$$

In order to find the center and radius, we need to change the equation of the circle into standard form, $(x-h)^2 + (y-k)^2 = r^2$. In order to get the equation into standard form, we have to complete the square with respect to both variables.

Grouping x's and y's together and moving the constant to the right side, we get

$$(x^2 + 24x) + (y^2 + 10y) = -160$$

Completing the square requires us to take the coefficient on the first degree terms, divide them by 2, then square the result before adding the result back to both sides.

The coefficient on the x term is 24, so

$$\frac{24}{2} = 12 \quad \rightarrow \quad 12^2 = 144$$

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The coefficient on the y term is 10, so

$$\frac{10}{2} = 5 \quad \rightarrow \quad 5^2 = 25$$

So we add 144 inside the parentheses with the x terms, 25 inside the parenthesis with the y terms and also add 144 and 25 to the right side with the -160.

$$(x^2 + 24x + 144) + (y^2 + 10y + 25) = -160 + 144 + 25$$

$$(x + 12)^2 + (y + 5)^2 = 9$$

Therefore, the center of the circle is at $(h, k) = (-12, -5)$ and its radius is $r = \sqrt{9} = 3$.

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